PULSATING COMBUSTION IN LARGE SYSTEMS

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A characteristic equation is derived for the excitation of pulsating combustion in equipment of blast-furnace air-heater type with two-pipe fuel and air supply.

Pulsating combustion occurs in various large industrial systems such as boilers and furnaces; serious difficulties and even damage can arise during use from such combustion in the air heaters of blast furnaces, for example.

We have examined the excitation conditions for such combustion in blast heaters at various plants [1-3]; we found that the pulsations arise from acoustic excitation of parts of the air system. Most often, the combustion chamber is excited at the very low frequency of 5-6 Hz in the style of a gigantic organ tube of height up to 35 m and diameter up to 2.5 m. The acoustic oscillations are maintained by the periodic supply and combustion of the gas–air mixture, with a favorable delay between the fuel input and combustion, this being close to a half-cycle, which thus satisfies Rayleigh's criterion. The mode of excitation is most similar to that in the familiar flickering-flame phenomenon.

However, when one applies the theories on such flames [4, 5], it is found that the excitation regions for the blast heaters do not always coincide with the derivation from the theory. For this reason we have considered a more general excitation scheme with a two-type gas and air supply.

Figure 1 shows a scheme for the system with three separate tubes: the combustion chambers 1, gas pipe 2, and air pipe 3, which are loaded at the ends by the referred acoustic impedances z_i, which incorporate the effects of adjacent parts (the space under the dome, the gas mixers, the combustion chamber proper, and so on), which are not themselves incorporated in the calculation scheme. The combustion zone and burner lie at the points where all three elements join ($x = 0$). It is assumed that the oscillations are simplified only by the variable heat release from the flame [3]. We neglect the speed of the gas by comparison with the speed of sound and also the damping of the sound along the elements. The small-perturbation method is applied [6].

The boundary conditions at the ends of the tubular elements are put in the following form:

$$\text{at } x = l_1: \delta p_1 = z_1 \delta v_1;$$

$$\text{at } x = -l_2: \delta p_2 = -z_2 \delta v_2;$$

$$\text{at } x = -l_3: \delta p_3 = -z_3 \delta v_2.$$

The steady-state resistance of the burner is

$$p_i - p_1 = k_i \frac{\rho_i v_i^2}{2},$$

and we obtain the boundary conditions at the junction between the elements for small perturbations:

$$\text{at } x = 0 \ \ \delta p_2 - \delta p_1 = k_2 \rho_2 \delta v_2;$$

$$\delta p_3 - \delta p_1 = k_3 \rho_3 \delta v_3.$$

The resistance coefficients k_i include the resistance of the throttle before the burner or flow control, and these quantities are considered as purely real.

We represent the flame for a gas–air mixture of heat of combustion q as a mobile surface of arbitrary shape of mean area F and mean propagation speed u, with heat transfer from this to the gas in the

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steady state (adiabatic conditions) in the form

$$Q = quF. \tag{3}$$

We assume that the volume of the burnt gases is equal to the volume of the combustion mixture $uF = V_c$, and that the flame speed and surface are dependent on the air-consumption factor α and various other conditions (flow turbulence, flame stabilization, etc.), which under otherwise identical conditions are determined by the flow speed or gas–air combustion mixture flow rate V_c. As the variations in u and F are linked, we assume that

$$u = f(\alpha); \; F = f(V_c). \tag{4}$$

We also assume that

$$q = f(\alpha). \tag{5}$$

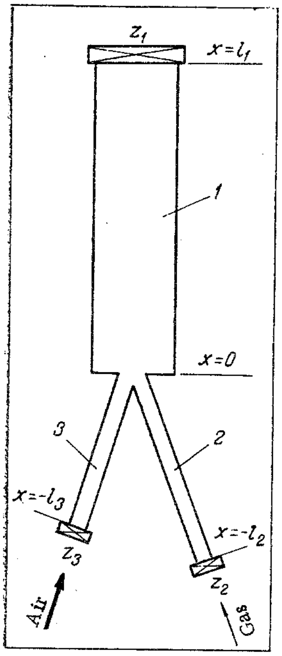

Fig. 1. The system.

The variable heat release from the flame under perturbed conditions is obtained from (3) and (4) and (5):

$$\delta Q = quF\left[\left(\frac{\partial \ln q}{\partial \ln \alpha} + \frac{\partial \ln u}{\partial \ln \alpha}\right)\frac{\delta\alpha}{\alpha} + \frac{\partial \ln F}{\partial \ln V_c}\cdot\frac{\delta V_c}{V_c}\right]. \tag{6}$$

We use the expression

$$\alpha = \frac{V_3}{V_0 V_2}, \; V_c = V_2 + V_3$$

with identical delays τ between gas and air supply and combustion, which enables us to express the perturbations in the volume flow rates before and after the combustion zone in terms of the heat-release perturbation. We follow Raushenbakh [6] in assuming the length of the combustion zone as substantially less than the wavelength, and neglect this in writing the acoustic relationships. Then we get for x = 0 that

$$F\delta v_1 - (F_2\delta v_2 + F_3\delta v_3) = F_2E_2\delta v_2 + F_3E_3\delta v_3, \tag{7}$$

where

$$E_2 = N_2\exp(i\varepsilon\tau), \; E_3 = N_3\exp(i\varepsilon\tau)$$

are the transfer functions of the combustion zone, which define the effects of the heat supply to the gas, and

$$N_2 = Sq\,[L - H(1 + \alpha V_0)]; \; N_3 = Sq\left[H\left(1 + \frac{1}{\alpha V_0}\right) + L\right]$$

are the moduli of the transfer functions of the moduli of the heat supply:

$$H = \frac{\partial \ln q}{\partial \ln \alpha} + \frac{\partial \ln u}{\partial \ln \alpha}; \quad L = \frac{\partial \ln F}{\partial \ln V_c}.$$

The quantity S is a coefficient of proportionality between the heat supply to the gas and the expansion.

The following equations [7] describe the perturbation of the gas motion for small oscillations in all the tubular elements:

$$\delta p = \left[A\exp\left(i\varepsilon\frac{x}{c}\right) + B\exp\left(-i\varepsilon\frac{x}{c}\right)\right]\exp(-i\varepsilon t); \tag{8}$$

$$\delta v = \frac{1}{\rho c}\left[A\exp\left(i\varepsilon\frac{x}{c}\right) - B\exp\left(-i\varepsilon\frac{x}{c}\right)\right]\exp(-i\varepsilon t), \tag{9}$$

$$\varepsilon = \omega + i\nu; \tag{10}$$

where

$$\omega = 2\pi f. \tag{11}$$

We substitute (8) and (9) into (1), (2), and (7) to get six homogeneous linear equations for the arbitrary constants A_i and B_i:

$$A_1(1 - \bar{z}_1)\exp\left(i\varepsilon\frac{l_1}{c_1}\right) = -B_1(1 + \bar{z}_1)\exp\left(-i\varepsilon\frac{l_1}{c_1}\right),$$

$$
\begin{aligned}
&A_2(1+\bar{z}_2)\exp\left(-i\varepsilon\frac{l_2}{c_2}\right)=-B_2(1-\bar{z}_2)\exp\left(i\varepsilon\frac{l_2}{c_2}\right),\\
&A_3(1+\bar{z}_3)\exp\left(-i\varepsilon\frac{l_3}{c_3}\right)=-B_3(1-\bar{z}_3)\exp\left(i\varepsilon\frac{l_3}{c_3}\right),\\
&A_1+B_1=(1-\bar{\mu}_2)A_2+(1+\bar{\mu}_2)B_2,\\
&A_1+B_1=(1-\bar{\mu}_3)A_3+(1+\bar{\mu}_3)B_3,\\
&\frac{A_1-B_1}{h_1}=\frac{(1+E_2)(A_2-B_2)}{h_2}+\frac{(1+E_3)(A_3-B_3)}{h_3},
\end{aligned}
\tag{12}
$$

where

$$\bar{z}_i=\frac{z_i}{\rho_i c_i};\ \bar{\mu}_i=\frac{\mu_i}{\rho_i c_i};\ \mu_i=k_i\rho_i v_i;\ h_i=\frac{\rho_i c_i}{F_i}.$$

We solve (12) and introduce the symbols

$$\frac{1-\bar{z}_i}{1+\bar{z}_i}=\exp(-2\psi_i);\ \psi_i=\alpha_i-i\beta_i;$$

$$\frac{1-\bar{\mu}_i}{1+\bar{\mu}_i}=\exp(-2\varphi_i),$$

to get the characteristic equation

$$\frac{1+E_2}{h_2}\cdot\frac{\operatorname{ch}\varphi_2\operatorname{ch}\left[\alpha_2-i\left(\beta_2+\frac{\varepsilon l_2}{c_2}\right)\right]}{\operatorname{sh}\left[(\varphi_2+\alpha_2)-i\left(\beta_2+\frac{\varepsilon l_2}{c_2}\right)\right]}+\frac{1+E_3}{h_3}\cdot\frac{\operatorname{ch}\varphi_3\operatorname{ch}\left[\alpha_3-i\left(\beta_3+\frac{\varepsilon l_3}{c_3}\right)\right]}{\operatorname{sh}\left[(\varphi_3+\alpha_3)-i\left(\beta_3+\frac{\varepsilon l_3}{c_3}\right)\right]}=-\frac{1}{h_1}\operatorname{cth}\left[\alpha_1-i\left(\beta_1+\frac{\varepsilon l_1}{c_1}\right)\right]. \tag{13}$$

This equation enables us to define the oscillation frequencies, the stability limits, and the regions of excitation in the planes of the various parameters.

The excitation is dependent on the absolute magnitude of the heat release and on the phase shift between that release and the pressure oscillations at the bottom of the combustion chamber; this shift, in turn, consists of two components: the phases for the supply of successive batches of the gas–air mixture and the delay between this supply and the combustion. The first component is determined by the acoustic features of the combustion chamber and supply lines (gas pipe and air pipe), while the second is governed by processes related to mixing and combustion. The pulsations may be excited or suppressed only for certain relations between these components, which in general are determined by (13). There are major difficulties in deriving a general solution to the characteristic equation, and we derive the excitation conditions for certain particular cases in order to compare the solutions with experiment.

The air section of the blast heater consists of a fan connected directly to the burner with a short air nozzle, whose length is much less than the acoustic wavelength. This has little effect on the excitation, and the long gas pipe is the main factor here. Similar conditions are produced in flickering-flame experiments by using a single gas-supply tube. Therefore, to compare the results with experiment we consider a one-pipe system containing a gas line.

In that case, the dimensionless resistance coefficient of the burner is $\bar{\mu}_2 = \operatorname{th}\varphi_2$, and we eliminate from (13) the term that incorporates the effect of the air line, which gives

$$\bar{\mu}_2+\operatorname{th}\left[\alpha_2-i\left(\beta_2+\frac{\varepsilon l_2}{c_2}\right)\right]=-\frac{h_1}{h_2}(1+E_2)\operatorname{th}\left[\alpha_1-i\left(\beta_1+\frac{\varepsilon l_1}{c_1}\right)\right]. \tag{14}$$

As $\bar{\mu}_2$ is real, the stability limit (for $\nu = 0$) will be satisfied by values of the natural frequencies ω such that $\bar{\mu}_2$ is real; these values are found from the condition

$$\operatorname{Im}(\bar{\mu}_2)=0. \tag{15}$$

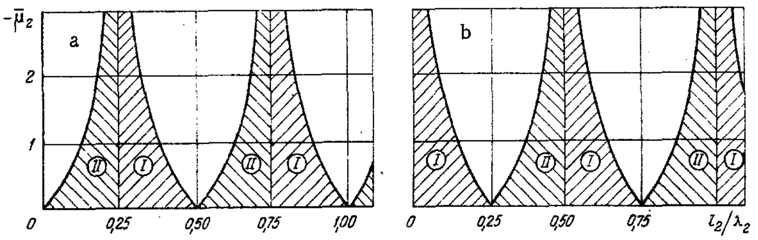

Fig. 2. Boundaries and regions of pulsation (hatched) in relation to relative gas-pipe length l_2/λ_2 and dimensionless burner resistance coefficient $\bar{\mu}_2$: a and b) gas inlet acoustically open and closed, respectively; I and II) for positive and negative values, respectively, in (22).

We simplify the problem by assuming that $\alpha_1 = 0$, i.e., there is no wave damping in the combustion chamber; the values of ω are given by (15) from

$$\operatorname{tg}\left(\beta_1 + 2\pi\frac{l_1}{\lambda_1}\right) = \frac{h_2}{h_1(1+N_2\cos\omega\tau)}\cdot\frac{\sin 2\left(\beta_2 + 2\pi\frac{l_2}{\lambda_2}\right)}{\operatorname{ch} 2\alpha_2 + \cos 2\left(\beta_2 + 2\pi\frac{l_2}{\lambda_2}\right)}. \tag{16}$$

We substitute (16) into (14) to get an equation defining the position of the stability limits:

$$N_2\sin\omega\tau\sin 2\left(\beta_2 + 2\pi\frac{l_2}{\lambda_2}\right) - \bar{\mu}_2\left[\operatorname{ch}2\alpha_2 + \cos 2\left(\beta_2 + 2\pi\frac{l_2}{\lambda_2}\right)\right](1+N_2\cos\omega\tau) - \operatorname{sh}2\alpha_2(1+N_2\cos\omega\tau) = 0. \tag{17}$$

The rules of [8] indicate that the excitation conditions correspond to $>$ in place of equality in the equation, while pulsation suppression corresponds to $<$.

If the heat input is large ($N_2 \gg 0$) and $\alpha_2 = \bar{\mu}_2 = 0$, i.e., the burner has no resistance, and the conditions at the end of the gas pipe are ideal, the system is most similar to that for a flickering flame, and one gets only standing waves. Then (17) simplifies even further, and the excitation conditions take the form

$$\sin\omega\tau\sin 2\left(\beta_2 + 2\pi\frac{l_2}{\lambda_2}\right) > 0. \tag{18}$$

As the gas-pipe length is increased, one then gets periodic sequences of stable and unstable regions, which follow at each quarter wavelength. If the combustion delay is less than half a period (or $\sin\omega\tau > 0$) the oscillations will be suppressed for an acoustically open inlet ($\beta_2 = n\pi$) for

$$(2n+1)\frac{\lambda_2}{4} < l_2 < (2n+2)\frac{\lambda_2}{4},\quad n = 0,\ 1,\ 2,\ldots \tag{19}$$

and for a closed inlet ($\beta_2 = (n + 1/2)\pi$) for

$$2n\frac{\lambda_2}{4} < l_2 < (2n+1)\frac{\lambda_2}{4},\quad n = 0,\ 1,\ 2,\ldots \tag{20}$$

The lengths of gas pipe resulting in excitation are obtained if these relationships are interchanged.

If the combustion delay is greater than half a period but less than one period (or $\sin\omega\tau < 0$), we get excitation for pipe lengths given by (19) and (20), while the lengths resulting in suppression are obtained if these are interchanged.

The relationships for the lengths of (19) and (20) have been derived as a particular case from the excitation condition of (17), and they agree with those indicated by Rayleigh [9]; they were derived by experiment by Sondhauss and derived theoretically in another way by Putnam [4] for a classical flickering flame. Pariel et al. [10] checked these relationships by experiment and recommended the choice of pipe lengths for industrial air heaters; however, on the blast heaters we have examined, the relative lengths of the gas pipes were within the recommended limits, but strong pulsations occurred. The recommendations were also found not to apply during a check at a Japanese plant [11].

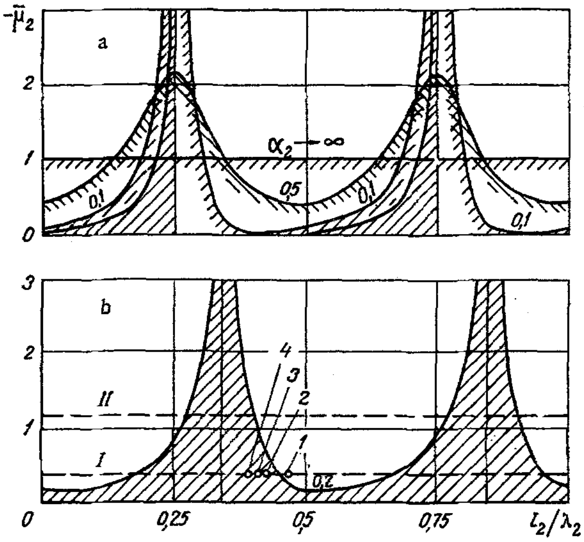

Fig. 3

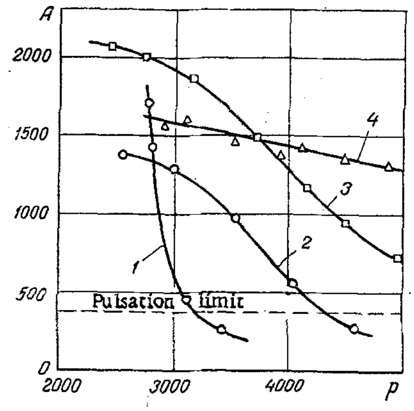

Fig. 4

Fig. 3. Boundaries and regions of pulsation (hatched) in relation to relative gas-pipe length l_2/λ_2 and burner resistance (incorporating gas-pipe input impedance): a) α_2 increasing (numbers on curves); b) for averaged air-heater working conditions; I) mean $\bar{\mu}_2$ for their heaters; II) the same on increasing the burner resistance by 2500 N/m^2; 1-4) relative gas-pipe lengths of air heaters.

Fig. 4. Effects of pressure p (N/m^2) in gas ahead of burner on the amplitude A (N/m^2) of pulsations at constant gas and air flow rates: 1-4) air heaters for which the relative gas-pipe lengths are given in Fig. 3.

If the burner resistance is incorporated for ideal conditions at the inlet $\alpha_2 = 0, \bar{\mu}_2 \neq 0$ equation (17) takes the following form for excitation:

$$\bar{\mu}_2 > \frac{N_2 \sin \omega\tau}{1 + N_2 \cos \omega\tau} \operatorname{tg}\left(\beta_2 + 2\pi \frac{l_2}{\lambda_2}\right). \tag{21}$$

The sign of the following quantity influences the way the pipe length and inlet acoustic conditions control the instability:

$$\frac{N_2 \sin \omega\tau}{1 + N_2 \cos \omega\tau}, \tag{22}$$

this determining the relationship between the modulus of the heat supply and the combustion delay.

The acoustic-energy loss is high in blast heaters, so we assume $N_2 \to \infty$, and get $\tan\omega\tau$ instead of (22).

Figure 2 shows the form taken by the instability regions in the plane of $\bar{\mu}_2$ and l_2/λ_2 for various signs for (22); Figure 2a shows regions constructed for acoustically open inlets to the gas pipe ($\beta_2 = n\pi$), while Fig. 2b shows the same for acoustically closed inlets ($\beta_2 = (n + 1/2)\pi$).

Increase in gas-pipe length results in alternation of stable and unstable regions, no matter whether the sign in (22) is positive or negative and whether the inlet is acoustically open or closed; the unstable regions become narrower as the burner resistance increases, while the stable regions expand. The effects of the resistance are dependent on the relative pipe length. Only very high burner resistances result in suppression for the acoustically open case for certain relative lengths, (l_2/λ_2 = 0.25; 0.75; 1.25...) and the same applies to the closed case (l_2/λ_2 = 0.5; 1.0; 1.5...).

These conclusions on the effect of burner resistance agree with the earlier results of [5] on the excitation conditions for flickering flames; however, the resulting instability regions do not correspond either to the pulsation conditions in blast heaters.

If we incorporate wave damping ($\alpha_2 \neq 0$) and the phase change at the impedance when the end of the gas pipe differs from ideal ($\beta_2 \neq n\pi/2$), the results are found to change substantially; the damping in the gas pipe results in traveling waves accompanying the standing ones, whose proportion is determined by α_2, and for $\alpha_2 \to \infty$ there are only traveling waves.

Figure 3a shows how the stability regions defined by (17) are distorted as α_2 increases; an ideal open acoustic input to the gas pipe has been assumed, with a delay representing the average for the blast heaters examined, $\omega\tau = 170°$; for $\alpha_2 = 0$ we get pulsation regions (hatched) similar to those given above (Fig. 2a, regions II).

If α_2 increases for $|\mu_2|$ small, the instability regions expand, while the stable ones contract, and at a certain damping level ($\alpha_2 \approx 0.1$ for the $\omega\tau$ assumed) and small $|\mu_2|$ the oscillations arise for any pipe length. Maxima and minima in the excitation occur at certain pipe lengths, and these are displaced by a quarter wavelength for an acoustically closed end. The positions of the minima are displaced towards larger pipe lengths as α_2 increases. When one incorporates the wave damping along the tube, the heights of the successive peaks decrease (those of the minima increase).

The values of α_2 for the maxima and minima come together as $|\mu_2|$ increases, and for $\alpha_2 \to \infty$ the maxima and minima vanish, with the excitation being determined by a straight line. At the minima, excitation is possible for $|\mu_2|$ small and impossible for $|\mu_2| \to 1$, i.e., when the resistance coefficient of the burner approaches the wave impedance of the median (ρc). This result substantially revises Jones' data [12] on oscillation in a flickering flame with damping in the supply tubes, and also the theoretical results of [4]. In the first case, the conclusion was that a flickering flame is excited at all tube lengths, probably for small burner resistances, while the resistance of the burner was not incorporated in the theoretical analysis of the second at all.

When the phase change is incorporated ($\beta_2 \neq n\pi$) for the input impedance of the gas pipe, the excitation regions in Fig. 3a are displaced to the right on account of the deviation from ideal conditions at the open end; in the limit, the displacement is a quarter wavelength (acoustically closed inlet).

Figure 3b shows the pulsation regions and the stable region for a blast heater for average parameters found by experiment ($\omega\tau = 170°$; $\alpha_2 = 0.2$ and $\beta_2 = 0.8\pi$); the actual acoustic characteristics of the gas pipe substantially alter the pulsation regions and shift them towards the regions of stable operation, as determined for ideal conditions at the end of the gas pipe.

Figure 3b shows also the relative gas-pipe lengths for the major blast heaters we examined; most such heaters lie in the pulsation zone (or near it for the average conditions).

The agreement between theory and experiment is confirmed by the effects of increasing the burner resistance (Fig. 4): the pulsations in the heater 1 were suppressed for a very low excess pressure (up to 500 N/m^2), while in heater 4 they persisted even at a considerable excess pressure (up to 2500 N/m^2). Although we have not incorporated the other individual features of the heaters, and the positions of the points are approximate, this behavior can be predicted approximately from Fig. 3b.

NOTATION

p	is the pressure;
v	is the flow velocity;
ρ	is the density;
c	is the speed of sound;
z	is the impedance;
λ	is the wavelength;
α, β, ψ	are the quantities representing the damping and the phase change at the impedance;
φ, μ	are the quantities representing the burner resistance;
k	is the resistance coefficient;
q	is the calorific value of an air–gas mixture;
u	is the mean flame speed;
F	is the area;
Q	is the heat produced by flame;
V	is the volume flow rate;
a	is the air consumption factor;
V_0	is the theoretical air needed for combustion of 1 m^3 of gas;
ω	is the angular fluctuation frequency;
f	is the fluctuation frequency;
ν	is the decrement;
t	is the time;
τ	is the combustion delay.

Subscripts and Superscripts

1, 2, and 3 refer to the combustion chamber, gas line, and air line;
c refers to the air–gas mixture.

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